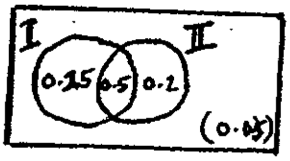
edex



# June 2005
# 6687 Statistics S5
# Mark Scheme

publication

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 1 (a) | $M'_X(t) = 13 + 198t + (1731t^2 +)\ldots$ | M′ and t=0 | M1 |
| | $E(X) = M'_X(0) = \underline{13}$ | 13 | A1 (2) |
| (b) | $M''_X(t) = 198 + (3462t) + \ldots$ | M″ | M1 |
| | $Var(X) = M''_X(0) - [M'_X(0)]^2$ | Formula and t=0 | M1 |
| | $= 198 - 169 = \underline{29}$ | 29 | A1 (3) |
| | | | Total 5 |
| 2 (a) |   | Method for 0.25 & 0.2 | M1 |
| | | Both correct | A1 |
| | P(Pass Exam) $= 0.5 + 0.6 \times 0.25 + 0.4 \times 0.2$ | | M1, M1A1ft |
| | $= 0.5 + 0.15 + 0.08$ | | |
| | $\underline{= 0.73}$ | 0.73 | A1 (6) |
| (b) | P(Fail I or Fail II \| Pass Exam) $= \dfrac{0.15 + 0.08}{0.73} = \underline{\dfrac{23}{73}}$ or AWRT 0.315 | | M1 A1ft A1ft A1 (4) |
| | | | Total 10 |
| 3 (a) | $G(1) = 1 \Rightarrow k[1 \times 5 + 2^4] = 1$ i.e. $k = \underline{\dfrac{1}{21}}$ (*) | | M1A1 (2) |
| (b) | $E(X) = G'_X(1)$.. $G'_X(t) = k[6t^2 + 12t^3 + 4(1+t)^3]$ | $G'_X(t)$ | M1 |
| | $G'_X(1) = k[6 + 12 + 4 \times 8]$ | $G'_X(1)$ | M1 |
| | $\therefore E(X) = \underline{\dfrac{50}{21}}$ or $2\frac{8}{21}$ or 2.38 (AWRT) | | A1 (3) |
| (c) | $G''_X(t) = k[12t + 36t^2 + 12(1+t)^2]$ | | M1 |
| | $G''_X(1) = 96k$ | $G''_X(1)$ | M1 |
| | $Var(X) = G''_X(1) + G'_X(1) - G'_X(1)^2$ | | M1 |
| | $= \dfrac{96}{21} + \dfrac{50}{21} - \dfrac{2500}{21^2} = \underline{\dfrac{566}{441}}$ or 1.28 (AWRT) | | A1 (4) |
| (d) | $P(X=3)$ = coefficient of $t^3$ in expansion of $G_X(t)$ | | |
| | $= k[2 + \binom{4}{3}]$, $= \dfrac{6}{21} = \underline{\dfrac{2}{7}}$ or $\underline{0.286}$ | | M1, A1 (2) |
| | | | Total 11 |

| Question Number | Scheme | | |
|---|---|---|---|
| 4 (a) | $X$ = no. of coughs in next 5 minutes $\quad X \sim Po(2)$ Implied | M1 | |
| | $P(X=3) = \frac{e^{-2}.2^3}{3!} = 0.1804\ldots \quad = \underline{0.180 \text{ or } 0.18 \text{ or } 0.1804}$ | A1 | (2) |
| (b) | $T$ = time between coughs $\quad T \sim Exp(2/5)$ Implied | M1 | |
| | $P(2 < T < 3) = \int_2^3 \frac{2}{5} e^{-\frac{2}{5}t}\,dt$ | | |
| | $= \left[-e^{-\frac{2}{5}t}\right]_2^3, = e^{-\frac{4}{5}} - e^{-\frac{6}{5}} = \underline{0.148}$ (or 0.15 or 0.1481) | M1, A1 | (3) |
| (c) | $(b)^2 \quad , = 0.02194\ldots \quad = \underline{0.022}$ or 0.0219 | M1, A1 | (2) |
| (d) | Probability is small so unlikely coughs at random or suggests there is some ground for the accusation | B1 | (1) |
| | | Total | (8) |
| 5 (a) | $M_X(t) = \frac{1/2}{\frac{1}{2} - t}$ or $\frac{1}{1-2t}$ | B1 | (1) |
| (b) | $M_Y(t) = M_{X_1}(t).M_{X_2}(t)\ldots M_{X_n}(t)$ | M1 | |
| | $= \left(\frac{1}{1-2t}\right)^4$ o.e. | A1 | (2) |
| (c) | $M_Y(t) = \frac{1}{(1-2t)^4}$ i.e. same form as $M_C(t)$ $\therefore Y \sim \chi^2$ | B1 | |
| | $\nu = 2m, \quad m = 4 \quad \therefore \quad \underline{\nu = 8}$ | B1 | (2) |
| (d) | Time to leave store $\sim \chi^2_8 \quad P(\chi^2_8 < 3.5) = 1 - 0.90 = \underline{0.10}$ | B1 | (1) |
| (e) | $P(\chi^2_8 > 20)$ cf $P(\chi^2_8 > 20.090) = 0.010 \therefore \underline{0.01}$ | B1 | (1) |
| | | Total | (7) |

| Question | Scheme | Marks |
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| 6 (a) | $X$ = no. of spin when 1st red occurs $\quad X \sim \text{Geo}(0.4)$ | M1 |
| | $P(X \leq 7) = 1 - P(X > 7) = 1 - (0.6)^7, \ \underline{= 0.972 \text{ 3dp}}$ (*) | M1, A1 (3) |
| (b) | $Y$ = no. of reds in 1st 7 spins $\quad Y \sim \text{Bin}(7, 0.4)$ Implied | M1 |
| | $P(Y = 3) = \binom{7}{3}(0.4)^3(0.6)^4$ (or Tables) $\underline{= 0.2903 \text{ or } 0.290 \text{ or } 0.29}$ | A1 (2) |
| (c) | $R$ = no. of spin on which 3rd red occurs $\quad R \sim \text{NegBin}(0.4, 3)$ Implied | M1 |
| | $P(R = 7) = \binom{6}{2}(0.4)^2(0.6)^4 \times 0.4, \ \underline{= 0.1244 \text{ or } 0.124}$ | M1, A1 (3) |
| (d) | $P(R \leq 7) = (c) + P(R=6) + P(R=5) + P(R=4) + P(R=3)$ | M1 |
| | $= (0.4)^3[15 \times (0.6)^4 + 10 \times (0.6)^3 + 6 \times (0.6)^2 + 3 \times 0.6 + 1]$ | M1 A1 |
| | $= 0.580096\ldots \quad \underline{= 0.58 \text{ or } 0.580 \text{ or } 0.5801}$ | A1 (4) |
| (e) | $P(A \text{ wins}) = 0.4 + (0.6)^3 \times 0.4 + (0.6)^6 \times 0.4 + \ldots$ i.e. G.P. | M1 |
| | $= \dfrac{0.4}{1 - (0.6)^3}, \ \underline{= 0.510 \text{ or } 0.51 \text{ or } 0.5102}$ Use of $S_\infty$ formula | M1, A1 (3) |
| (f) | $P(A \text{ wins} \mid X \leq 7) = \dfrac{(0.4 + (0.6)^3 \times 0.4 + (0.6)^6 \times 0.4)}{(a)}, \ \underline{= 0.5196 \text{ or } 0.520}$ | M1, A1 (2) |
| | | Total (17) |
| 7 (a) | $X \sim B(10, p)$. $P(\text{Accept}) = P(X \leq 1) + P(X=2) \times P(X=0)$ | M1 |
| (i) | $p = 0.10 \Rightarrow P(\text{Accept}) = 0.7361 + (0.9298 - 0.7361) \times 0.3487 = \underline{0.804}$ | M1 A1 |
| (ii) | $p = 0.15 \Rightarrow P(\text{Accept}) = 0.5443 + (0.8202 - 0.5443) \times 0.1969 = \underline{0.599}$ | A1 (4) |
| (b) | Expected proportion accepted is $P(\text{Accept}) \times p$ | M1 |
| | $= \underline{0.08036\ldots} \quad$ (ii) $= \underline{0.0897\ldots} \quad$ (both $< 9\%$) | A1, A1 (3) |
| (c) | Expected no. sampled is: $20 \times P(X=2) + 10[1 - P(X=2)]$ | M1 |
| | $p = 0.10 \quad = 20[0.9298 - 0.7361] + 10[0.8063] \underline{= 11.937\ldots \approx 12}$ | A1 cso (2) |
| (d) | $Y$ = no. of defectives. $Y \sim B(12, 0.10)$. Require $P(Y \leq c) < 0.90$ | |
| | $P(Y \leq 3) = 0.9744$, $P(Y \leq 2) \underline{= 0.8891}$, $\underline{\text{Accept if } Y \leq 2}$ | B1, B1 (2) |
| (e) | $P(\text{Acceptance}) = \underline{0.7358}$, $\therefore$ proportion accepted $\underline{= 0.11\ldots \approx 11\%}$ | M1, A1 (2) |
| (f) (i) | $\underline{p = 0.10}$ | |
| | P(acceptance) using single scheme is greater than double scheme | M1 (cf above) |
| | So use of double scheme a $\underline{\text{disadvantage}}$ cao | A1 |
| | $\underline{p = 0.15}$ | |
| | Expected proportion accepted using single scheme is $> 9\%$ | M1 (cf above) |
| | So use of double scheme is an $\underline{\text{advantage}}$ cao | A1 (4) |
| | | Total (17) |